Question 10 (9 marks)

(a) Draw the subset of the complex plane determined by on the axes below.

(3 marks)

<EFOFEX>
id:fxd{83c81f21-553b-46d0-bb68-ea3effab2e03}

FXData:

</EFOFEX>

<EFOFEX>
id:fxd{3b95f65c-8585-4808-a8f7-0c5259a934c2}

FXData:

</EFOFEX>

(b) The circular arc in the diagram represents  
the locus of a complex number .  
  
  
Without using or , write  
equations or inequalities in terms of   
for the indicated locus.  
  
  
  
 (3 marks)

(c) Describe the subset of the complex plane determined by .

(3 marks)

Question 10 (9 marks)

(a) Draw the subset of the complex plane determined by on the axes below.

(3 marks)

<EFOFEX>
id:fxd{ae2acad5-552d-4117-9965-f12397a8d54b}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See diagram |
| Specific behaviours |
| ✓ indicates points in plane  ü draws perp’ bisector with dotted line  ü shades correct region |

<EFOFEX>
id:fxd{3b95f65c-8585-4808-a8f7-0c5259a934c2}

FXData:

</EFOFEX>

(b) The circular arc in the diagram represents  
the locus of a complex number .  
  
  
Without using or , write  
equations or inequalities in terms of   
for the indicated locus.  
  
  
  
 (3 marks)

|  |
| --- |
| Solution |
| Circle has centre and radius . |
| Specific behaviours |
| ✓ indicates correct centre and radius  ü writes inequality for circle  ü writes restriction for |

(c) Describe the subset of the complex plane determined by .

(3 marks)

|  |
| --- |
| Solution |
| Distance between and in complex plane is .  Hence must lie on the line segment between and inclusive in the complex plane.  Alternatively, when then locus is . |
| Specific behaviours |
| ✓ indicates distance between points  ü indicates subset is a line segment  ü correct description that includes endpoints |

Question 16 (8 marks)

(a) Determine all solutions to the equation in exact polar form. (3 marks)

(b) Consider the ninth roots of unity expressed in polar form .

(i) Determine the roots for which . (2 marks)

(ii) Use all nine roots to show that .

(3 marks)

Question 16 (8 marks)

(a) Determine all solutions to the equation in exact polar form. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ expresses in polar form  ü states one correct solution  ü states all correct solutions |

(b) Consider the ninth roots of unity expressed in polar form .

(i) Determine the roots for which . (2 marks)

|  |
| --- |
| Solution |
| Hence |
| Specific behaviours |
| ✓ general expression for roots  ü correct roots |

(ii) Use all ten roots to show that .

(3 marks)

|  |
| --- |
| Solution |
| The nine roots are given by , and the sum of these roots, and hence their real parts, will be :  But and . Hence |
| Specific behaviours |
| ✓ uses sum of real parts of all roots is  ü uses and known values  ü simplifies to obtain required result |

Question 18 (9 marks)

Let and .

(a) Determine an exact value for

(i) . (1 mark)

(ii) . (1 mark)

(b) Let , where is a positive integer.  
Determine the minimum value of so that is purely imaginary. (3 marks)

The modulus of complex number is and its argument is , where .

(c) Determine the value of for which

(i) is minimum. (1 mark)

(ii) is maximum, where . (3 marks)

Question 18 (9 marks)

Let and .

(a) Determine an exact value for

(i) . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

(ii) . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

(b) Let , where is a positive integer.  
Determine the minimum value of so that is purely imaginary. (3 marks)

|  |
| --- |
| Solution |
| For then . |
| Specific behaviours |
| ✓ expression for  ü indicates values of for  ü correct value of |

The modulus of complex number is and its argument is , where .

(c) Determine the value of for which

(i) is minimum. (1 mark)

|  |
| --- |
| Solution |
| For to be minimum, and must be parallel but in opposite direction.  Hence |
| Specific behaviours |
| ✓ correct value |

(ii) is maximum, where . (3 marks)

|  |  |
| --- | --- |
| Solution | |
| Locus of is circle, centre and radius .  Maximum , and from geometric considerations this occurs when . | <EFOFEX> id:fxd{172deb0d-9f3d-4abd-917e-38f0b01e4f40}  FXData:  </EFOFEX> |
| Specific behaviours | |
| ✓ sketch diagram (possibly seen in part(b)(i))  ü indicates for maximum argument  ü correct value | |

Question 10 (7 marks)

(a) Solve the equation , giving exact solutions in the form .

(4 marks)

(b) One solution of the equation , where is a positive integer, is .  
If solutions of the equation satisfy , determine, with reasoning, the least value of . (3 marks)

Question 10 (7 marks)

(a) Solve the equation , giving exact solutions in the form .

(4 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ writes in polar form with correct modulus  ü determines correct argument  ü states one correct solution  ü states all correct solutions |

(b) One solution of the equation , where is a positive integer, is .  
If solutions of the equation satisfy , determine, with reasoning, the least value of . (3 marks)

|  |
| --- |
| Solution |
| Solutions to the equation must be of the form . Noting that before simplification the multiple of will always be even, then the given solution can be written as and hence minimum value of .  With this value of and , then and so least value of . |
| Specific behaviours |
| ✓ indicates general solution for roots of unity  ü deduces value of  ü states correct number of solutions with required argument |

Question 14 (8 marks)

(a) On the Argand planes below sketch the locus of the complex number given by

(i) . (3 marks)

<EFOFEX>
id:fxd{8f6b7c62-5104-4e7c-a808-145927697574}

FXData:

</EFOFEX>

(ii) . (3 marks)

<EFOFEX>
id:fxd{8f6b7c62-5104-4e7c-a808-145927697574}

FXData:

</EFOFEX>

(b) For the locus in part (a), determine the minimum value  
for . (2 marks)

Question 14 (8 marks)

(a) On the Argand planes below sketch the locus of the complex number given by

(i) . (3 marks)

<EFOFEX>
id:fxd{418065f0-c170-4b92-95a0-d4be7dae08a9}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See diagram. |
| Specific behaviours |
| ü plots both points  ü sketches perpendicular bisector  ü correct axis intercepts |

(ii) . (3 marks)

<EFOFEX>
id:fxd{dd45aafa-65c8-4795-afe5-f3462013e4bc}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See diagram. |
| Specific behaviours |
| ✓ deals with conjugate  ü indicates a shaded circle  ü correct centre and radius |

(b) For the locus in part (a), determine the minimum value  
for . (2 marks)

|  |
| --- |
| Solution |
| Shortest distance from (on axis) to line.  Hence minimum is . |
| Specific behaviours |
| ✓ indicates perpendicular distance to line  ü correct minimum value |

Question 17 (5 marks)

The complex number is shown on the Argand diagram below and .

<EFOFEX>
id:fxd{c1640ed4-4c7d-481f-b8b4-1c0c6f628650}

FXData:

</EFOFEX>

(a) Describe the geometric transformation performed by when another complex number is multiplied by it, and plot and label on the Argand diagram. (2 marks)

(b) Plot and label the complex number on the Argand diagram. (3 marks)

Question 17 (5 marks)

The complex number is shown on the Argand diagram below and .

<EFOFEX>
id:fxd{5652a50f-c2d2-4692-a759-5408b75a3c64}

FXData:

</EFOFEX>

(a) Describe the geometric transformation performed by when another complex number is multiplied by it, and plot and label on the Argand diagram. (2 marks)

|  |
| --- |
| Solution |
| will rotate another complex number clockwise by about the origin (or rotate about the origin). |
| Specific behaviours |
| ü correctly describes transformation  ✓ correctly locates on diagram |

(b) Plot and label the complex number on the Argand diagram. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct argument of  ü indicates correct argument of reduced to  ü correctly locates on diagram |



































